FACULTY OF ECONOMICS AND BUSINESS - ZAGREB

INVESTMENT ANALYSIS

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Total holding period return (total profitability)	$R_t = \frac{D_t + (P_t - P_0)}{P_0} = \frac{D_t + P_t}{P_0} - 1$ $R_t = \frac{I_t + (B_t - B_0)}{B_0} = \frac{I_t + B_t}{B_0} - 1$ $R_t = y_d + \%\Delta P$ $R_t = y_b + \%\Delta B$ $R_{0-n} = (1+R_1)\cdot(1+R_2)\cdot\dots\cdot(1+R_n) - 1$ $R_{0-n}^{annual} = \sqrt[n]{(1+R_1)\cdot(1+R_2)\cdot\dots\cdot(1+R_n)} - 1$ $R_t^{FX} = \frac{(D_t + P_t)\cdot(1+a)}{P_0} - 1$ $R_t^{portfolio} = \sum_{j=1}^{n} w_j \cdot R_{t_j}$	R_t – total holding period return D_t I_t – current income P_t B_t – price of investment at the end of the period P_0 B_0 – price of investment at the beginning of the period y_d y_b – current yield (%) %ΔP %ΔB – capital gain / loss (%) R_{0-n} – total profitability (holding period more than a year) R^{annual}_{0-n} – average annual holding period return (average annual profitability) R^{FX}_t – total profitability (return) of investments denominated in foreign currency a – appreciation of currency (%) $R_t^{port.}$ – total portfolio return
Real / nominal rate of return	$r = \frac{1+n}{1+i} - 1$ $r \approx n - i$	r – real rate of return n – nominal rate of return i – inflation rate
Measures of portfolio return	$Gross\ return = \frac{\text{Increase in the value of a portfolio} + \text{Dividends, interests etc.}}{\text{Value of a portfolio at the beginning of the period}}$ $Net\ return = \frac{\text{Increase in the value of a portfolio} + \text{Dividends, interests etc.} - \text{Management and custody costs}}{\text{Value of a portfolio at the beginning of the period}}$ $\text{Net return after taxes} = \frac{\text{Net increase in the value of a portfolio} + \text{Dividends, interests etc. after taxes} - \text{Management and custody costs}}{\text{Value of a portfolio at the beginning of the period}}$ $\text{Real net return after taxes} = \frac{1 + \text{Net return after taxes}}{1 + \text{Increase in the price level}} - 1$	
Return and risk of an investment	$E(r)_j = \sum_{i=1}^{m} E(r)_{ij} \cdot p_i$ $Var(r)_j = \sigma^2(r)_j = \sum_{i=1}^{m} \left[E(r)_{ij} - E(r)_j\right]^2 \cdot p_i$ $\sigma(r)_j = \sqrt{Var(r)_j}$ $V(r)_j = \frac{\sigma(r)_j}{E(r)_j}$	$E(r)_j$ – expected return of investment j $Var(r)_j$ – variance of investment j $\sigma(r)_j$ – standard deviation of investment j $V(r)_j$ – coefficient of variation of investment j p_i – probability of scenario i (m – number of possible scenarios) $E(r)_{ij}$ – expected return of investment j in scenario i

Return and risk of a risky portfolio	$E(r)_P = \sum_{j=1}^{n} E(r)_j \cdot w_j$ $E(r)_P = \sum_{i=1}^{m} E(r)_{iP} \cdot p_i$ $Cov_{xy} = \sum_{i=1}^{m} [E(r)_{ix} - E(r)_x] \cdot [E(r)_{iy} - E(r)_y] \cdot p_i$ $Cov_{xy} = \sigma_x \cdot \sigma_y \cdot \rho_{xy}$ $\rho_{xy} = \frac{Cov_{xy}}{\sigma_x \cdot \sigma_y}$ $Var(r)_P = \sigma^2(r)_P = w_x^2\sigma_x^2 + 2w_xw_yCov_{xy} + w_y^2\sigma_y^2$ $Var(r)_P = \sigma^2(r)_P = \sum_{i=1}^{m} [E(r)_{iP} - E(r)_P]^2 \cdot p_i$ $\sigma(r)_P = \sqrt{Var(r)_P}$	$E(r)_P$ – expected return of a protfolio w_j – weight of investment j in a portfolio (n – number of investments in a portfolio) $E(r)_{iP}$ – expected return of a portfolio in scenario i Cov_{xy} – covariance between returns of two investments x and y ρ_{xy} – correlation coefficient between returns of two investments x and y $Var(r)_P$ – variance of a portfolio's return $\sigma(r)_P$ – standard deviation of a portfolio's return
Return and risk of a total portfolio	$E(r)_{TP} = E(r)_P \cdot w_P + r_F \cdot w_F$ $E(r)_{TP} = r_F + w_P \cdot [E(r)_P - r_F]$ $\sigma(r)_{TP} = w_P \cdot \sigma(r)_P$	$E(r)_{TP}$ – expected return of a total portfolio r_F – return of a risk-free asset w_P – weight of a risky portfolio in a total portfolio w_F – weight of a risk-free asset in a total portfolio $\sigma(r)_{TP}$ – standard deviation of a total portfolio's return
Coupon bond	$B_0 = I_t \cdot IV_{k_b}^t + N \cdot II_{k_b}^t \qquad I_t = i \cdot N$ $G.F. \qquad k_b \approx \frac{I_t + \frac{N - B_0}{t}}{0{,}6 \cdot B_0 + 0{,}4 \cdot N}$ $M.I.S. \quad exact\ calculation\ of\ k_b\ (iteration)$	B_0 – value / price of a bond k_b – required rate of return on a bond(yield to maturity) N – nominal value t – maturity i – coupon / nominal interest rate I_t – coupon (interest) A_t – annuity
Zero-coupon bond	$B_0 = N \cdot II_{k_b}^t \qquad k_b = \sqrt[t]{\frac{N}{B_0}} - 1$	
Annuity bond	$B_0 = A_t \cdot IV_{k_b}^t \qquad A_t = N \cdot V_i^t \qquad IV_{k_b}^t = \frac{B_0}{A_t}$	
Yield to call (coupon bond)	$B_0 = I_t \cdot IV_{k_c}^{t_c} + B_c \cdot II_{k_c}^{t_c} \qquad B_c = N \cdot (1 + p_c)$ $G.F. \qquad k_c \approx \frac{I_t + \frac{B_c - B_0}{t_c}}{0{,}6 \cdot B_0 + 0{,}4 \cdot B_c}$ $I.R.M. \quad exact\ calculation\ of\ k_c\ (iteration)$	B_c – call price k_c – yield to call t_c – years when bond becomes callable p_c – call premium (%)
Duration	$\tau = \frac{\sum_{t=1}^{T} \frac{t \cdot V_t}{(1 + k_b)^t}}{B_0}$ $\tau = \frac{\frac{1 \cdot I_t}{(1 + k_b)^1} + \frac{2 \cdot I_t}{(1 + k_b)^2} + \cdots + \frac{T \cdot (I_t + N)}{(1 + k_b)^T}}{B_0}$ $\tau^m = \frac{\tau}{1 + k_b}$ $\%\Delta B_0 = -\tau \cdot \frac{\Delta k_b}{1 + k_b} = -\tau^m \cdot \Delta k_b$ $\tau_P = \sum_{j=1}^{n} \tau_j \cdot w_j$	τ – duration of a bond V_t – cash flows of a bond τ^m – modified duration of a bond $\%\Delta B_0$ – percentage price change of a bond Δk_b – yield to maturity change (in percentage points) τ_P –duration of a portfolio (that is compound of bonds)
Preffered shares (constant dividends)	$P_0 = \frac{D_t}{k_s}$ $k_s = \frac{D_t}{P_0}$	P_0 – share price D_t – preffered dividends per share k_s – required rate of return

Common shares (constant growth of dividends)	$G.M. \qquad P_0 = \frac{D_0 \cdot (1+g)}{k_s - g} = \frac{D_1}{k_s - g}$ $k_s = \frac{D_0 \cdot (1+g)}{P_0} + g = \frac{D_1}{P_0} + g$ $P_t = \frac{D_t \cdot (1+g)}{k_s - g} \qquad P_t = P_0 \cdot (1+g)^t$ $D_t = D_0 \cdot (1+g)^t \qquad E_t = E_0 \cdot (1+g)^t$ $D_t = E_t \cdot d \qquad d = 1 - z$ $P/E = \frac{P_t}{E_t} = \frac{d \cdot (1+g)}{k_s - g}$	P_0 – share price D_0 – payed-out dividends per share D_1 – expected dividends per share k_s – required rate of return g – growth rate of dividends (earnings, share price) P_t – share price in period t D_t – expected dividends per share in period t E_t – expected earnings per share in period t d – payout dividend ratio z – retained earnings ratio P/E – price-earnings ratio g_s – supernormal growth rate of dividends g_n –normal growth rate of dividends T – period of linear decrease of growth rate VP_0 – value of the company $FCFF_0$ – free cash flow to the firm k – weighted average cost of capital N_s – number of the common shares $FCFEps_0$ – free cash flow to the equity (per share)
Common shares (variable dividends, supernormal growth, H-model)	$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_s)^t}$ $P_0 = \sum_{t=1}^{T} \frac{D_t}{(1+k_s)^t} + \frac{P_T}{(1+k_s)^T}$ $P_0 = \sum_{t=1}^{T} \frac{D_0 \cdot (1+g_s)^t}{(1+k_s)^t} + \frac{D_T \cdot (1+g_n)}{(k_s - g_n) \cdot (1+k_s)^T}$ $P_0 = \frac{D_0 \cdot (1+g_n)}{k_s - g_n} + \frac{D_0 \cdot H \cdot (g_s - g_n)}{k_s - g_n} \qquad H = \frac{T}{2}$	
Common shares (free cash flow)	$VP_0 = \frac{FCFF_0 \cdot (1+g)}{k - g}$ $P_0 = \frac{Value\ of\ common\ equity._0}{N_s} = \frac{VP_0 - Liabilities_0}{N_s}$ $P_0 = \frac{FCFEps_0 \cdot (1+g)}{k_s - g}$	
Market multipliers	$P/E = \frac{PPS}{EPS} \qquad P/D = \frac{PPS}{DPS}$ $P/S = \frac{PPS}{SPS} \qquad P/B = \frac{PPS}{BPS}$ $EPS = \frac{Net\ income}{N_s} \qquad DPS = \frac{Dividends}{N_s}$ $SPS = \frac{Sales}{N_s} \qquad BPS = \frac{Book\ value\ of\ equity}{N_s}$ $N_s = number\ of\ issued\ shares - number\ of\ treasury\ shares$ $PPS_j = (P/E)_s \cdot EPS_j \qquad PPS_j = (P/D)_s \cdot DPS_j$ $PPS_j = (P/S)_s \cdot SPS_j \qquad PPS_j = (P/B)_s \cdot BPS_j$	P/E – price-earnings ratio P/D – price-dividends ratio P/S – price-sales ratio P/B – price to book value of equity ratio PPS – market price of share EPS – earnings per share DPS – dividends per share SPS – sales (revenue) per share BPS – book value of equity per share N_s – number of outstanding shares PPS_j –price of j company $(P/X)_s$ – standard price to selected variable ratio

CAPM	$k_{S_i} = k_F + \beta_i \cdot (k_M - k_F)$ $\beta_i = \frac{Cov_{iM}}{\sigma_M^2}$ $\sigma_P = \sqrt{\beta_P^2 \sigma_M^2 + \sigma_{\varepsilon P}^2} \approx \beta_P \sigma_M$	k_S – required rate of return k_F – nominal risk-free interest rate β – systematic risk k_M –return of market index Cov_{iM} – covariance between a specific security's return and market portfolio return σ_M – standard deviation of a market portfolio return σ_P – standard deviation of a well diversified portfolio's return $\sigma_{\varepsilon P}$ – standard deviation of a residual (specific risk) of a well diversified portfolio
Margin account (long position)	$i.m.\% = \frac{Purchase\ amount - Loan}{Purchase\ amount} == \frac{Initial\ margin}{Purchase\ amount}$ $a.m.\% = \frac{Actual\ market\ value\ of\ a\ portolio - Loan}{Actual\ market\ value\ of\ a\ portolio}$ $c.m.\% = \frac{Minimum\ market\ value\ of\ a\ portolio - Loan}{Minimum\ market\ value\ of\ a\ portolio} =$ $= \frac{Maintainance\ margin}{Maintainance\ margin + Loan}$ $r.m.\% = \frac{P_{t+1} - P_t + D_t - k_d \cdot (1 - i.m.\%) \cdot P_t}{i.m.\% \cdot P_t}$	i.m.% – initial margin requirement (%) a.m.% – actual margin in investors account (%) c.m.% – maintainance margin (%) r.m.% – investor's return on investment (short sale) over the margin account (%) P_t – share price in period t D_t – dividends per share k_d – interest rate on margin loan / deposit (%)
Margin account (short sale)	$i.m.\% = \frac{Initial\ margin}{Short\ sale\ value}$ $a.m.\% = \frac{In.margin + Short\ sale\ value - Market\ value\ of\ securities}{Market\ value\ of\ securities}$ $c.m.\% = \frac{In.margin + Short\ sale\ value - Max.market\ value\ of\ securities}{Max.market\ value\ of\ securities}$ $r.m.\% = \frac{P_t - P_{t+1} - D_t + k_d \cdot i.m.\% \cdot P_t}{i.m.\% \cdot P_t}$	
Income, savings, investment and consumption	$Y_0 = C_0 + S_0 + I_0$ $k_S = k_F + k_R$ $C_0 = Y_0 + \frac{S_1}{1 + k_F}$ $C_1 = Y_1 + S_0(1 + k_F)$ $maxC_0 = Y_0 - I_0 + \frac{Y_1}{1 + k_F} + \frac{I_1}{1 + k_S}$	Y_t – income in period t C_t – consuming in period t S_t – savings in period t I_t – investment in period t k_F – risk-free interest rate k_S – risk adjusted discount rate